V2, 27.10.2022
OE2IGL
V3, 16.11.2022
V4, 17.12.2022
V5, 5.8.2023
V6, 3.9.2023
V7, 9.12.2023

Antenna Basics

General form of Directivity

$$\boxed{D = \frac{4\pi}{\Omega}}$$

J. Kraus: $\Omega_{\square} = \Theta_V \cdot \Theta_H$ $\quad D = \frac{4\pi}{\Theta_V \Theta_H} = \frac{4\pi \cdot 180^2}{\pi^2 \Theta_V^\circ \Theta_H^\circ} = \frac{41253}{\Theta_V^\circ \Theta_H^\circ}$

$\Omega_{\circ} = \frac{\pi}{4} \Theta_V \Theta_H$ $\quad D = \frac{4 \cdot 4}{\pi \Theta_V \Theta_H} = \frac{16 \cdot 180^2}{\pi^2 \Theta_V^\circ \Theta_H^\circ} = \frac{52525}{\Theta_V^\circ \Theta_H^\circ}$

valid for HPBW > 56°

Tai & Pereira: (C. Balanis p.54)

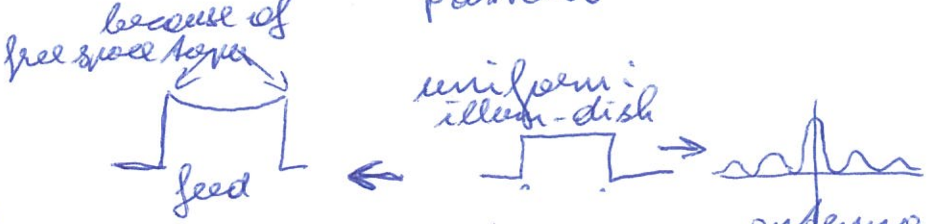
$$D = \frac{32 \ln 2}{\Theta_V^2 + \Theta_H^2} = \frac{16 \ln 2}{\Theta^2} = \frac{16 \ln 2 \cdot 180^2}{\Theta^{\circ 2} \pi^2} = \frac{36407}{\Theta^{\circ 2}}$$

valid for HPBW < 56°
(large apertures)

Beam factor, HPBW, illumination taper

$$\boxed{\text{HPBW } \Theta_{-3dB} = b \cdot \frac{\lambda}{d}}$$

b ... beam factor
describes illumination pattern



Jacob W.M. Baars:

power pattern $g(u) = \left[\frac{4}{1+\tau}\left(\tau \frac{J_1(u)}{u} + 2(1-\tau)\frac{J_2(u)}{u^2}\right)\right]^2$

$u = \frac{\pi d}{\lambda} \sin\Theta$

$$\boxed{b = 1{,}269 - 0{,}566 \cdot \tau + 0{,}534\tau^2 - 0{,}208\tau^3}$$

illuminated dish

edge Taper $T = 20 \log \tau$

e.g. $T = -10\ \text{db} \Rightarrow b = 1{,}13\cancel{7}$

$T = 0\ \text{db} \Rightarrow b = 1{,}029$

$T \Rightarrow -\infty \Rightarrow b = 1{,}269$

HPBW: $\theta^{\circ}_{-3db} = b \cdot \frac{180}{\pi} \cdot \frac{\lambda}{d} = b \cdot 57{,}3\,\frac{\lambda}{d}$

$T = 0\ \text{db} \Rightarrow b = 1{,}029 \Rightarrow \theta^{\circ}_{-3db} = 58{,}96 \cdot \frac{\lambda}{D}$ ... uniform illumination

$T \Rightarrow -\infty\ \text{db} \Rightarrow b = 1{,}269 \Rightarrow \theta^{\circ}_{-3db} = 72{,}7 \cdot \frac{\lambda}{d}$

$\Rightarrow$ b defines factor (60 ... 70).

Gain

Tai & Perreira:

$$\underset{\substack{\uparrow\\ \text{0 db Taper} \Rightarrow \text{uniformly illuminated}}}{D} = \frac{16 \ln 2}{\theta^2} = \frac{16 \ln 2 \cdot d^2}{b_0^2\ \lambda^2} = \frac{16 \ln 2}{1{,}029^2}\left(\frac{d}{\lambda}\right)^2 = 10{,}5\left(\frac{d}{\lambda}\right)^2$$

$$\approx \pi^2\left(\frac{d}{\lambda}\right)^2 = \left(\frac{\pi d}{\lambda}\right)^2$$

$$G_{(taper)} = \frac{16 \ln 2 \cdot d^2}{b^2 \cdot \lambda^2} = \frac{16 \cdot \ln 2 \cdot d^2}{b^2 \cdot \lambda^2}\left(\frac{b_0}{b_0}\right)^2 = \frac{16 \ln 2\, d^2}{b_0^2\ \lambda^2}\left(\frac{b_0}{b}\right)^2$$

$$= \left(\frac{\pi d}{\lambda}\right)^2 \cdot \underbrace{\left(\frac{1{,}029}{b}\right)^2}_{\eta_i\ =\ \eta_T \cdot \eta_S}$$

$$\boxed{G_{(taper),db} = \underbrace{20 \log \frac{\pi d}{\lambda}}_{D_0 \ldots \text{uniform illumination}} + \underbrace{20 \log \frac{1{,}029}{b(taper)}}_{\text{"taper" \& spill over loss of non-uniform illumination}}} + \underbrace{\text{add. losses}}_{\text{see page 3}}$$

e.g.: $T = -10\ \text{db} \Rightarrow b = 1{,}135$

$$\left(\frac{1{,}029}{1{,}135}\right)^2 = 0{,}822 \mathrel{\hat{=}} 82{,}2\,\% = \eta_i$$

All additional losses (RUZE surface, blockage feed, spillover, ohmic, polarisation, phase focus, ..) reduce further the gain

$\boxed{\text{Taper ---- efficiency}}$ η_t ... C. Balanis or g_i (Milligan T.)

feed W. M. Boas / T. Milligan p. 194 : gaussian distribution

$$\boxed{\eta_t = \frac{2(1-e^{-\alpha})^2}{\alpha(1-e^{-2\alpha})}}$$

$$\alpha = \frac{\text{Taper}}{20} \cdot \ln 10$$

$$(\text{Taper} = 8{,}686 \cdot \alpha)$$

e.g.: edge taper = −10 db

$\Rightarrow \alpha = 1{,}151$

$\eta_t = 0{,}902 \Rightarrow \text{loss} \approx 9{,}8\,\%$

T. Milligan, p. 385 : $\cos^{2N}(\varphi/2)$ distribution

$$\eta_t = \frac{4(N+1)(1-u^N)^2}{N^2\left(1-u^{2(N+1)}\right)\left(\tan\left(\frac{\varphi_0}{2}\right)\right)^2}$$

$$u = \cos\left(\frac{\varphi_0}{2}\right)$$

$$\varphi_0 = 2\arctan\left(\frac{1}{4(f/D)}\right)$$

R. C. Hansen : „A one-parameter circular aperture distribution with narrow beam-width and low sidelobes"

uses Bessel function.

e.g.: edge taper = −10 db

$\Rightarrow h = 2{,}5$

$\Rightarrow \eta_t = 0{,}908 \Rightarrow \text{loss} \approx 9{,}2\,\%$

Spillover efficiency

gaussian distribution: no source, my own consideration analogy $\cos^N$ distribution

$$\boxed{\eta_s = 1 - e^{-2k}}$$

$\cos^{2N}(\psi/2)$ distribution: T. Milligan

$$\eta_s = 1 - u^{2(N+1)}$$

e.g.: gauss edge taper = −10db
$\rightarrow k = 1{,}151$
$\rightarrow \eta_s = 1 - e^{-2 \cdot 1{,}151}$
$\eta_s = 0{,}9 \mathrel{\hat=} 90\%$

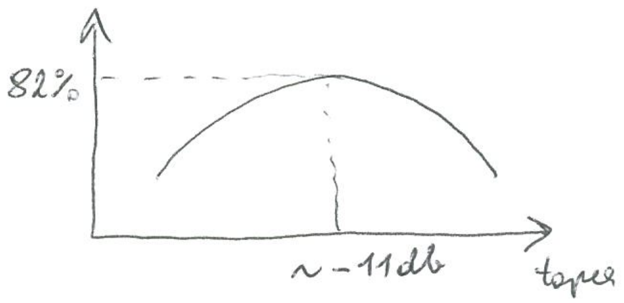
Illumination efficiency

$$\boxed{\eta_i = \eta_t \cdot \eta_s}$$

η_t and η_s are coupled. Both efficiencies depend on feed pattern and edge taper of dish.

$\eta_i = 65\% \ldots 70\%$ is possible

~82% max. theoretical efficiency at taper ≈ −11 db



My Gibertini offset dish (1m) claims an illumination efficiency of 70% with matched feed. Specified gain is calculated gain with taper of −10db.

Blockage efficiency

J. Baars, p. 86:

$$\boxed{\eta_B = \left(1 - \frac{1}{\eta_i}\left(\frac{d_B}{d}\right)^2\right)^2}$$

d_B -- blocking diameter
d -- dish diameter

$$\text{loss}_{B,db} = 10 \log \eta_B$$

Surface efficiency

Ruze:

$$\boxed{\eta_{surface} = e^{-\left(\frac{4\pi RMS}{\lambda}\right)^2}}$$

$\frac{\text{highest top} - \text{lowest valley}}{3} \approx RMS$

$$\text{loss}_{,surface,db} = -685\left(\frac{RMS}{\lambda}\right)^2 \left(= 10 \log e(1) \cdot \left(-\frac{4\pi RMS}{\lambda}\right)^2\right)$$

Focus efficiency

due to axial misalignment of feed.

J. W. M. Baars:

T. Milligan: z -- misalignment

$$S = \frac{z}{\lambda}\left(1 - \cos\left(2 \arctan \frac{1}{4(f/D)}\right)\right), \quad \alpha = \frac{\text{Taper}_{,db}}{8{,}686}$$

$$\boxed{\text{Phase Error efficiency} = \frac{\alpha^2\left(1 - 2e^{-\alpha}\cos(2\pi S) + e^{-2\alpha}\right)}{\left(\alpha^2 + (2\pi S)^2\right)\left(1 - e^{-\alpha}\right)^2}}$$

$$\text{loss}_{,phase,db} = 10 \log \text{PE efficiency}$$

e.g.: $f/D = 0{,}6$ $\qquad k = \frac{12}{8{,}686} = 1{,}38$

Taper = 12 db

47 GHz ≙ 6,37 mm

$z = 1{,}27$ mm $(0{,}2 \cdot \lambda)$

$$S = 1 \cdot \left(1 - \cos\left(2 \arctan \frac{1}{4 \cdot 0{,}6}\right)\right) = 0{,}3$$

$$PEloss = 10 \log \frac{1{,}38^2 \left[1 - 2e^{-1{,}38} \cdot \cos\left(2\pi\, 0{,}3 \frac{180}{\pi}\right) + e^{-2 \cdot 1{,}38}\right]}{(1{,}38^2 + 4\pi^2 \cdot 0{,}3^2)(1 - e^{-1{,}38})^2}$$

$PEloss = -1{,}2$ db

$f/D = 0{,}35$

$S = 0{,}68 \Rightarrow PEloss = -6{,}7$ db

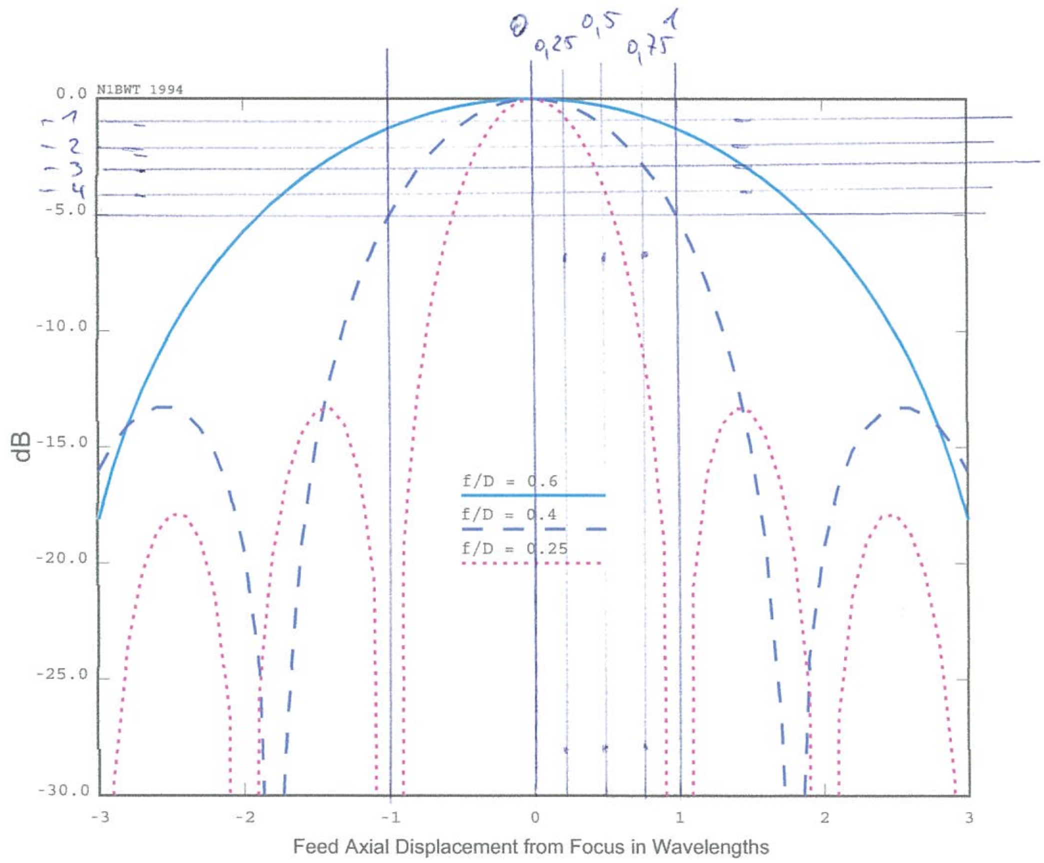


Figure 4-9. Loss due to Axial Feed Displacement from Focus

Antenna efficiency

Sometimes also called „aperture efficiency"
if $\eta_{polarisation} \cdot \eta_B \cdot \eta_{sur} \cdot \eta_{focus} \approx 1$ (losses negliable)

$$G_{ant} = \underbrace{D \cdot \underset{\approx 1,0}{\eta_{ohmic}} \cdot \overbrace{\eta_t \cdot \eta_s}^{\eta_i} \cdot \underset{\approx 0,95-0,99}{\eta_{polarisation}} \cdot \eta_B \cdot \eta_{sur} \cdot \eta_{Focus}}_{\eta_{ant}}$$

$$G_{ant,dB} = 10 \log D + 10 \log \eta_{ohmic} + 10 \log \eta_i + \ldots$$

$T_{spillover}$

$$T_{spillover,K} = \left\{1 - \arccos\left[\frac{\tan(Ele - offset)}{\tan\left(2 \arctan \frac{1}{4(f/D)}\right)}\right] \cdot \frac{1}{\pi}\right\} \cdot (273{,}15 + Temp) \cdot (1 - \eta_s) + \left[\arccos \frac{\tan(Ele - offset)}{\tan\left(2 \arctan \frac{1}{4(f/D)}\right)}\right] \cdot \frac{1}{\pi} \cdot T_{sky} \cdot (1 - \eta_s)$$

T_{sky}

$$T_{sky,K} = \left(1 - 10^{\frac{-A_{zen}}{10 \sin Ele}}\right) \cdot \left[0{,}81(273{,}15 + Temp) + 37{,}4\right] + T_{CMB} \cdot 10^{\frac{-A_{zen}}{10 \sin Ele}}$$

Planck radiation:

Rayleigh-Jeans approximation only valid if

$\frac{h\nu}{kT} << 1$ otherwise $T' = \frac{h\nu}{k(e^{\frac{h\nu}{kT}}-1)}$

$h = 6{,}62 \cdot 10^{-34}$

$k = 1{,}38 \cdot 10^{-23}$

$T_{CMB} = 2{,}73\,K$ needs to be corrected at higher µwave frequencies using T'

Also for any temperature, e.g. T_{sky}, ..

example: 47 GHz

$T = 23K \rightarrow T' = 21{,}5\,K$

$T = 100K \rightarrow T' = 98{,}7\,K$

$\boxed{G/T}$

$$\boxed{G/T_{db} = G_{ant,db} - 10\log(T_{RCV} + T_{spill} + T_{sky})}$$

$T_{RCV} = (10^{\frac{NF}{10}} \cdot 290) - 290$

$\boxed{\text{Main beam Efficiency}}$

Jacob W.M. Baars, p 114-124

$$\cancel{\eta_{antenna}} = k \cdot \eta_{MB}$$

$$\eta_{MB} = \frac{\Omega_{MB}}{\Omega_{ant}} = \frac{\Omega_{MB} \cdot A \cdot \eta_{ant}}{\lambda^2}$$

$$= \frac{\Omega_{MB} \cdot d^2 \pi}{4 \lambda^2} \cdot \eta_{ant}$$

$$\Omega_{MB} = \frac{\pi}{4 \ln 2} \Theta^2$$

$$= 1{,}133\, \Theta^2$$

$$\Rightarrow \eta_{MB} = \frac{\pi \cancel{d^2}}{4} \cdot \frac{1{,}133}{\cancel{\lambda^2}} \cdot b^2 \cdot \frac{\cancel{\lambda^2}}{\cancel{d^2}} \cdot \eta_{Ant}$$

$$\boxed{\eta_{MB} = 0{,}8899 \cdot b^2 \cdot \eta_{Ant}}$$

e.g.: Taper $= -10\,\mathrm{dB} \Rightarrow b = 1{,}135$

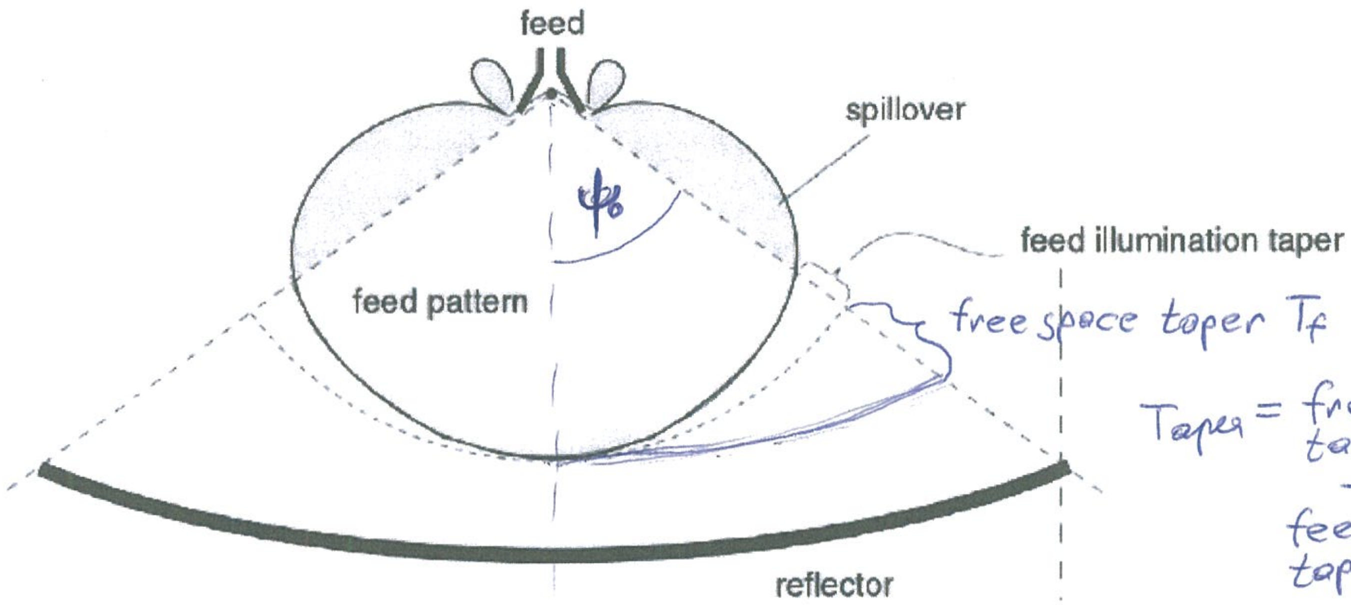
$$\eta_{MB} = 1{,}15 \cdot \eta_{Ant}$$

$$\Rightarrow \eta_{MB} = 1{,}15 \cdot 06 = 0{,}69$$

$$\eta_{MB} = 1{,}15 \cdot 0{,}7 = 0{,}80$$

Edge Taper Feed-Dish Prime-Focus

Spillover and amplitude efficiency



Taper = free space taper + feed illumi taper

$$T_f = 20\log\left[1+\left(\frac{1}{4\,f/d}\right)^2\right]$$

$$\varphi_0 = 2\arctan\left(\frac{1}{4\,f/d}\right)$$

$$T_f = 20\log\left(1+\sin^2\frac{\varphi_0}{2}\right)$$

$$T_f = -20\log\left(\cos\varphi_0/2\right)$$

example: $f/d = 0.66$, $T = -10\,db$

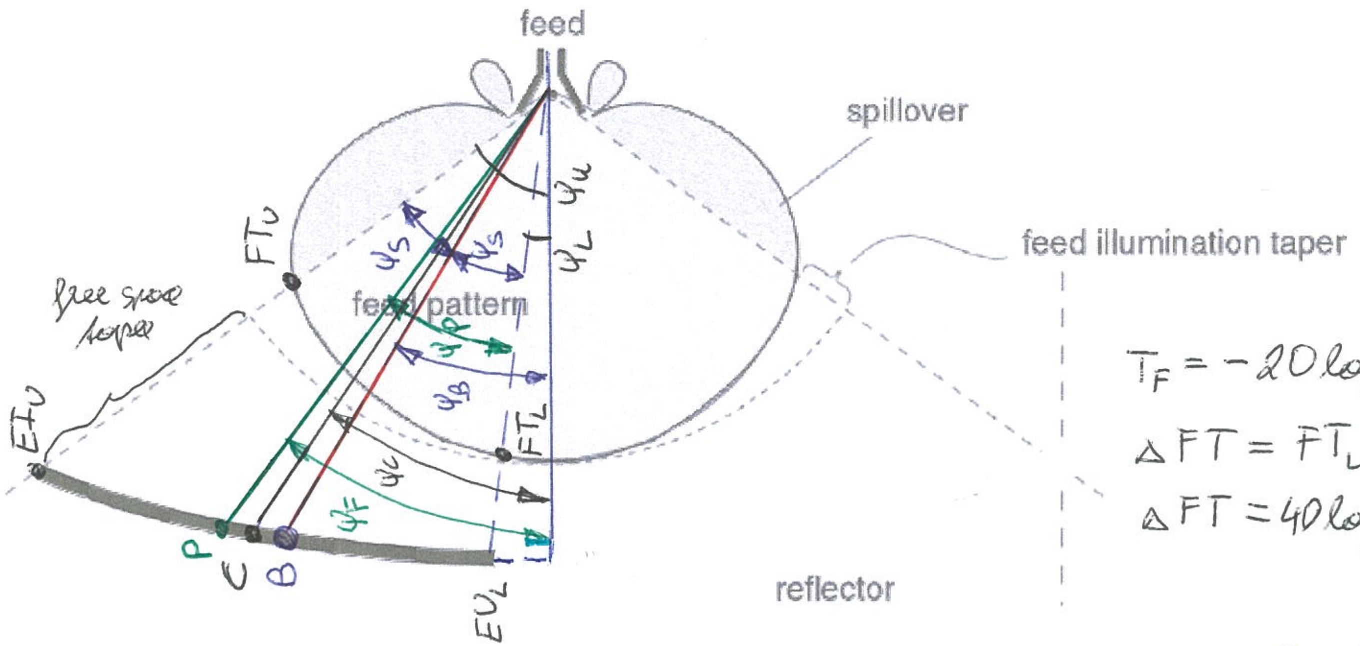
$$\varphi_0 = 2\arctan\left(\frac{1}{4\cdot 0.66}\right) = 41{,}5^\circ$$

$$T_f = 20\log\left(1+\left(\frac{1}{4\cdot 0.66}\right)^2\right) = 1{,}2\,db$$

$$T_{feed} = -10+1{,}2 = -8{,}8\,db$$

Offset

Spillover and amplitude efficiency



$$T_F = -20\log\left(\cos^2\frac{\psi}{2}\right)$$

$$\Delta FT = FT_L - FT_u$$

$$\Delta FT = 40\log\left(\frac{\cos\psi_L/2}{\cos\psi_u/2}\right) + \underset{0}{\underset{\shortparallel}{\Delta EI}}$$

rotate feed to ψ_F, free space taper + illum taper must be equal at lower and upper rim.

(11)

Milligan
Stutzman

$$\psi_S = \tan^{-1} \frac{8 \cdot F \cdot D}{16 \cdot F^2 + 4H^2 - D^2}$$

$$\psi_B = \tan^{-1} \frac{16 \cdot F \cdot H}{16 \cdot F^2 + D^2 - 4H^2}$$

$$\psi_L = \psi_B - \psi_S \qquad \psi_U = 2 \cdot \psi_S + \psi_L$$

$$\psi_C = 2 \cdot \tan^{-1} \frac{H}{2F} \qquad \psi_p = \psi_f - \psi_L$$

↳ for gaussian beam $\psi_f = \psi_C$

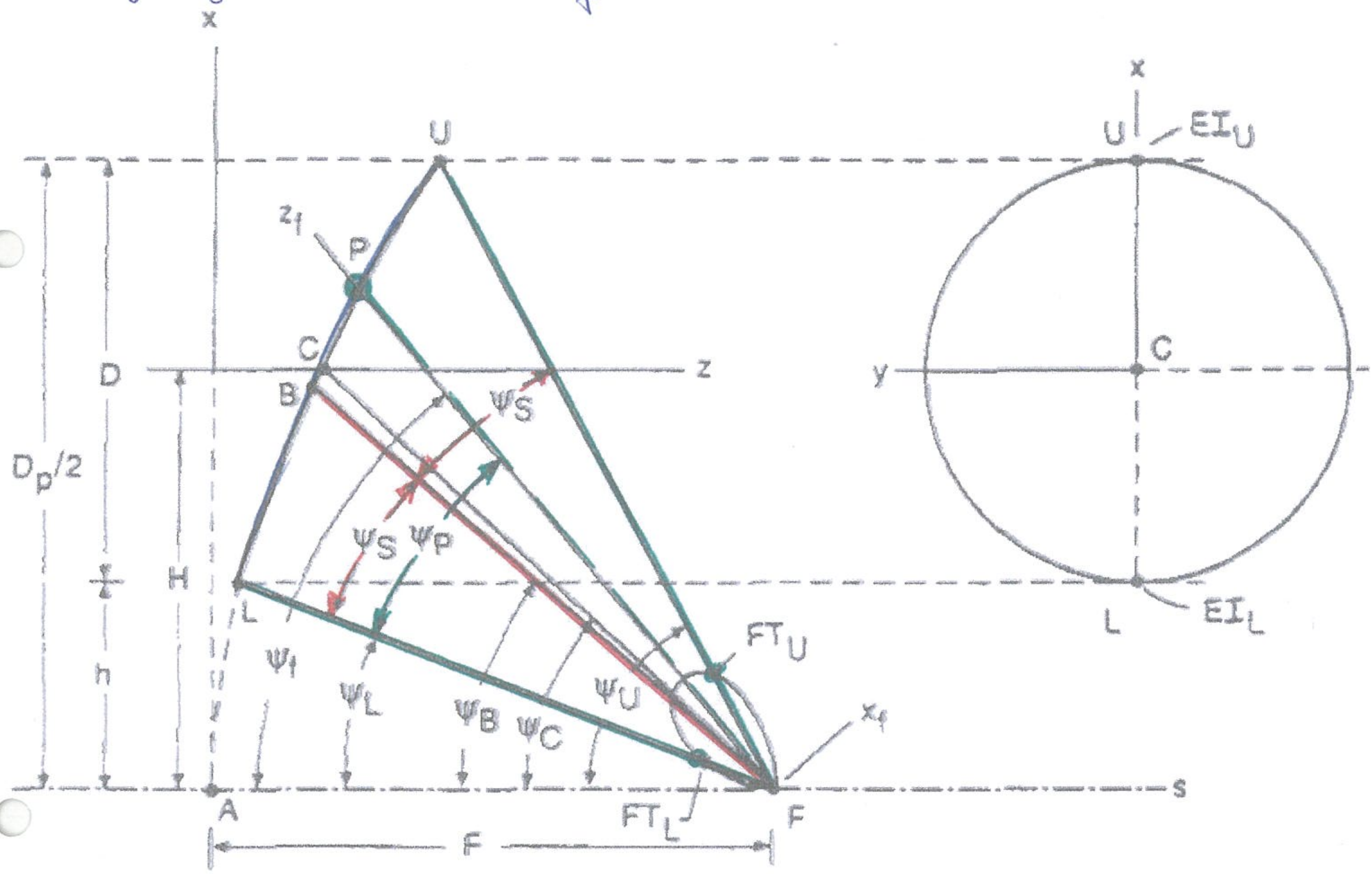


Figure 1. The geometry for the offset parabolic reflector. See Table 1 for the definitions of the parameters.

spherical spreading loss: $SPL(\psi) = -20 \log\left(\cos^2 \frac{\psi}{2}\right)$

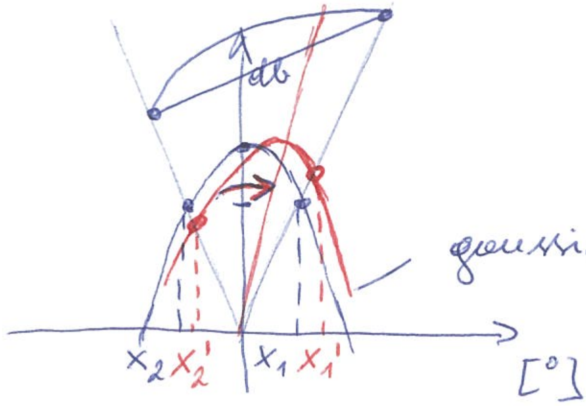
design equation:

$$\measuredangle FT = \underbrace{FT_L - FT_U}_{\text{edge taper}} = 40 \log\left(\frac{\cos \frac{\psi_L}{2}}{\cos \frac{\psi_U}{2}}\right) + \Delta EI$$

↳ soll 0 sein

10.12.2022
OE2IGL

Offset antenna and feed direction



gaussian feed beam $f(x) = e^{-\frac{2.77 x^2}{HPBW^2}}$

$$\Delta FT = 40 \log \frac{\cos \varphi_L/2}{\cos \varphi_U/2}$$

$$\Delta FT = f(x_1') - f(x_2') = 10 \log e^{-\frac{2.77 \cdot x_1'^2}{HPBW^2}} - 10 \log e^{-\frac{2.77 \cdot x_2'^2}{HPBW^2}}$$

with $x_1 = -x_2$, $x_1 - x_2 = BW = \varphi_S \cdot 2$

$x_1' - x_2' = BW = \varphi_S \cdot 2$

$$= -10 \frac{2.77 \cdot x_1'^2}{HPBW^2} \log e(1) + 10 \frac{2.77 (x_1' - BW)^2}{HPBW^2} \cdot \log e(1)$$

$$\Delta FT = -\frac{12.04}{HPBW^2} (\cancel{x_1'^2} - \cancel{x_1'^2} + 2x_1' BW - BW^2)$$

$$\Rightarrow x_1' = \frac{-\Delta FT \cdot HPBW^2}{2 \cdot 12.04 \cdot BW} + \frac{BW}{2} \quad \Rightarrow \varphi_P = \frac{\Delta FT \cdot HPBW^2}{24.08 \cdot BW} + \varphi_S$$

with:

$$f\left(\frac{BW}{2}\right)_{dB} = 10 \log e^{\frac{-2.77 \left(\frac{BW}{2}\right)^2}{HPBW^2}} = -\left(\text{Edge Taper} + \frac{\Delta FT}{2}\right)$$

$$-\underbrace{10 \log e(1) \cdot \frac{2.77}{4}}_{3} \cdot \frac{BW^2}{HPBW^2} = -\left(\text{Edge Taper} + \frac{\Delta FT}{2}\right)$$

$$HPBW^2 = \frac{3}{ET - \frac{\Delta FT}{2}} \cdot BW^2$$

$$\Rightarrow \varphi_P = \frac{\Delta FT \cdot 3 \cdot BW^{\cancel{2}}}{24.08 \; \cancel{BW} (ET - \Delta FT/2)} + \varphi_S$$

$$\boxed{\varphi_P = \frac{\Delta FT \cdot BW}{4.01 \cdot (2ET - \Delta FT)} + \varphi_S}$$

$$\varphi_F = \varphi_L + \varphi_P$$

e.g.: $\varphi_L = 0{,}7^\circ$ $\quad 2\varphi_S = 2(37{,}7-0{,}7) = 74^\circ = BW$

$\varphi_U = 37{,}7^\circ + 37^\circ$

$ET = 10\,db$

$$\Delta FT = 40 \log \frac{\cos 0{,}7/2}{\cos 37{,}7/2} = 3{,}99\,db$$

$$\varphi_{\cancel{B}} = \frac{\Delta FT \cdot BW}{4.01 \cdot (2ET - \Delta FT)} = \frac{3{,}99 \cdot 74}{4.01 \cdot (2 \cdot 10{,}0 - 3{,}99)} = 4{,}6^\circ$$

$$\underline{\varphi_F} = \varphi_P + \varphi_L = 4{,}6 + 37 + 0{,}7 = \underline{42{,}3^\circ}$$

$\varphi_B = 37 + 0{,}7 = 37{,}7^\circ$ (bisector)